Exercise 60
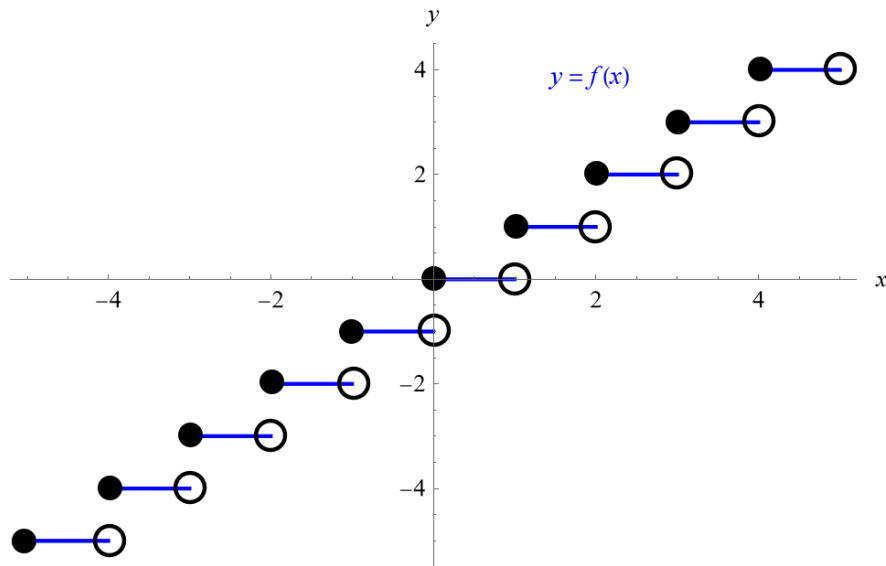
Where is the greatest integer function $f(x) = [\![x]\!]$ not differentiable? Find a formula for f' and sketch its graph.

Solution

A graph of the greatest integer function (also known as the floor function) is shown below.

The function is not differentiable at any of the integers because of the discontinuities.

$$f'(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Z} \\ \text{undefined} & \text{if } x \in \mathbb{Z} \end{cases}$$